

### 4.OA Operations and Algebraic Thinking

4.OA.A Use the four operations with whole numbers to solve problems.

**4.OA.A.1** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

**D.10** Compare numbers using multiplication

**4.OA.A.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

**D.11** Compare numbers using multiplication: word problems

**D.** Multiply 1-digit numbers by 2-digit numbers: word problems

**D.** Multiply 1-digit numbers by 3-digit or 4-digit numbers: word problems

**F.2** Comparison word problems: addition or multiplication?

**4.OA.A.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**D.31** Estimate products word problems: identify reasonable answers

**E.14** Divide 2-digit numbers by 1-digit numbers: interpret remainders

**E.20** Divide larger numbers by 1-digit numbers: interpret remainders

**F.6** Word problems with extra or missing information

**F.8** Multi-step word problems involving subtraction

**F.9** Multi-step word problems with strip diagrams

**F.11** Multi-step word problems

**F.12** Multi-step word problems involving remainders

- F.13** Multi-step word problems: identify reasonable answers
- G.4** Write variable equations to represent word problems
- Multi-step addition word problems

#### 4.OA.B Gain familiarity with factors and multiples.

**4.OA.B.4** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

- A.18** Prime and composite: up to 20
- A.19** Prime and composite: up to 100
- D.3** Choose the multiples of a given number up to 10
- D.7** Identify factors
- D.8** Choose numbers with a particular product
- D.9** Find all the factor pairs of a number

#### 4.OA.C Generate and analyze patterns.

**4.OA.C.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

- K.1** Find the next shape in a pattern
- K.3** Make a repeating pattern
- K.5** Use a rule to complete a number pattern
- K.6** What is true about the given pattern?
- K.7** What is true about the pattern made by the rule?
- K.8** Identify mistakes in number patterns
- K.13** Shape patterns

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## 4.NBT Number and Operations in Base Ten

## 4.NBT.A Generalize place value understanding for multi-digit whole numbers.

**4.NBT.A.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

- A.3** Value of a digit
- A.4** Relationship between place values

**4.NBT.A.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using  $>$ ,  $=$ , and  $<$  symbols to record the results of comparisons.

- A.1** Place value models
- A.2** Convert between standard and expanded form
- A.7** Place value review
- A.8** Writing numbers up to 1,000 in words: convert words to digits
- A.9** Writing numbers up to 1,000 in words: convert digits to words
- A.10** Writing numbers up to 100,000 in words: convert words to digits
- A.11** Writing numbers up to 100,000 in words: convert digits to words
- A.12** Writing numbers up to one million in words: convert words to digits
- A.13** Writing numbers up to one million in words: convert digits to words
- A.16** Spell word names for numbers up to one million
- A.25** Compare numbers up to one hundred thousand
- A.26** Compare numbers up to one million
- J.1** Place value word problems

**4.NBT.A.3** Use place value understanding to round multi-digit whole numbers to any place.

- A.21** Rounding: up to millions place
- A.22** Rounding input/output tables
- B.10** Estimate sums
- B.11** Estimate sums: word problems
- C.8** Estimate differences

- C.9 Estimate differences: word problems
- D.28 Estimate products: multiply by 1-digit numbers
- D.29 Estimate products: multiply by 2-digit numbers
- D.30 Estimate products: word problems
- E.25 Divide by 1-digit numbers: pick the better estimate
- F.5 Estimate sums, differences, products, and quotients: word problems

**4.NBT.B** Use place value understanding and properties of operations to perform multi-digit arithmetic.

**4.NBT.B.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.

- B.1 Add two numbers up to five digits
- B.2 Add two numbers up to five digits: word problems
- B.5 Addition: fill in the missing digits
- B.6 Properties of addition
- B.7 Add 3 or more numbers up to millions
- B.9 Choose numbers with a particular sum
- C.1 Subtract numbers up to five digits
- C.2 Subtract numbers up to five digits: word problems
- C.5 Subtraction: fill in the missing digits
- C.7 Choose numbers with a particular difference
- F. Comparison word problems with addition and subtraction
- F.15 Mentally add and subtract numbers ending in zeroes

**4.NBT.B.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- D.4 Multiplication facts to 12
- D.13 Multiply 1-digit numbers by 2-digit numbers using area models I
- D.14 Multiply 1-digit numbers by 2-digit numbers using area models II

- D.15 Multiply 1-digit numbers by 2-digit numbers
- D.16 Multiply 1-digit numbers by 3-digit or 4-digit numbers using area models I
- D.17 Multiply 1-digit numbers by 3-digit or 4-digit numbers using area models II
- D.18 Multiply 1-digit numbers by 3-digit or 4-digit numbers using expanded form
- D.20 Multiply 1-digit numbers by 3-digit or 4-digit numbers
- D.23 Multiplication patterns over increasing place values
- D.24 Properties of multiplication
- D.25 Distributive property: find the missing factor
- D.26 Multiply using the distributive property
- D.27 Use one multiplication fact to complete another
- D.32 Multiply 2-digit numbers by 2-digit numbers using area models I
- D.33 Multiply 2-digit numbers by 2-digit numbers using area models II
- D.34 Multiply 2-digit numbers by 2-digit numbers using partial products
- D.35 Box multiplication
- D.36 Lattice multiplication
- D.37 Multiply a 2-digit number by a 2-digit number: complete the missing steps
- D.38 Multiply a 2-digit number by a 2-digit number
- D.39 Multiply a 2-digit number by a 2-digit number: word problems
- D.43 Multiply numbers ending in zeroes
- D.44 Multiply numbers ending in zeroes: word problems

**4.NBT.B.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- D.22 Multiply by 10 or 100
- E.5 Properties of division
- E.6 Divide 2-digit numbers by 1-digit numbers using arrays
- E.7 Divide 2-digit numbers by 1-digit numbers using area models
- E.9 Divide using the distributive property
- E.11 Divide 2-digit numbers by 1-digit numbers

- E.12 Divide 2-digit numbers by 1-digit numbers: word problems
- E.13 Divide 2-digit numbers by 1-digit numbers: complete the table
- E.14 Divide 2-digit numbers by 1-digit numbers: interpret remainders
- E.15 Divide 3-digit numbers by 1-digit numbers using area models
- E.16 Divide using partial quotients
- E.17 Divide larger numbers by 1-digit numbers
- E.18 Divide larger numbers by 1-digit numbers: word problems
- E.19 Divide larger numbers by 1-digit numbers: complete the table
- E.20 Divide larger numbers by 1-digit numbers: interpret remainders
- E.21 Choose numbers with a particular quotient
- E.22 Division patterns over increasing place values
- E.23 Divide numbers ending in zeroes by 1-digit numbers

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## 4.NF Number and Operations—Fractions

### 4.NF.A Extend understanding of fraction equivalence and ordering.

**4.NF.A.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

- O.5 Find equivalent fractions using area models
- O.6 Graph equivalent fractions on number lines
- O.8 Equivalent fractions: find the missing numerator or denominator
- O.9 Fractions with denominators of 10 and 100

**4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

- O.15 Compare fractions using models
- O.16 Benchmark fractions

- O.17** Compare fractions using benchmarks
- O.18** Compare fractions using benchmarks: find the missing numerator
- O.19** Compare fractions
- O.20** Compare fractions: find the missing numerator or denominator
- O.21** Compare fractions in recipes

**4.NF.B** Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

**4.NF.B.3** Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .

**4.NF.B.3a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

- P.5** Add fractions with like denominators using area models
- P.6** Add fractions with like denominators using strip models
- P.8** Add fractions with like denominators
- P.9** Subtract fractions with like denominators using area models
- P.10** Subtract fractions with like denominators using strip models
- P.12** Subtract fractions with like denominators

**4.NF.B.3b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

- P.1** Decompose fractions into unit fractions using models
- P.2** Decompose fractions into unit fractions
- P.3** Decompose fractions
- P.4** Decompose fractions multiple ways

**4.NF.B.3c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

- P.7** Add fractions with like denominators using number lines

- P.11 Subtract fractions with like denominators using number lines
- P.13 Add and subtract fractions with like denominators using number lines
- P.19 Add and subtract mixed numbers with like denominators

**4.NF.B.3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

- P.16 Add and subtract fractions with like denominators: word problems
- P.17 Add and subtract fractions with like denominators in recipes
- P.20 Add and subtract mixed numbers with like denominators: word problems

**4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

**4.NF.B.4a** Understand a fraction  $a/b$  as a multiple of  $1/b$ .

- R.1 Multiply unit fractions by whole numbers using number lines
- R.2 Multiply unit fractions by whole numbers using models
- R.3 Multiples of unit fractions: find the missing numbers
- R.4 Multiply unit fractions by whole numbers: sorting

**4.NF.B.4b** Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number.

- R.5 Multiply unit fractions by whole numbers
- R.7 Multiply fractions by whole numbers using number lines
- R.8 Multiply fractions by whole numbers using models
- R.9 Multiply fractions by whole numbers using models: complete the equation
- R.10 Multiples of fractions: find the missing numbers
- R.11 Multiply fractions by whole numbers: sorting
- R.12 Multiply fractions by whole numbers

**4.NF.B.4c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

- R.6 Multiply unit fractions by whole numbers: word problems
- R.13 Multiply fractions by whole numbers: word problems
- R.14 Multiply fractions and mixed numbers by whole numbers in recipes

#### 4.NF.C Understand decimal notation for fractions, and compare decimal fractions.

**4.NF.C.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

- Q.1 Add fractions: denominators 10 and 100
- Q.2 Identify fraction expressions with a particular sum: denominators of 10 and 100

**4.NF.C.6** Use decimal notation for fractions with denominators 10 or 100.

- S.2 Model decimals and fractions
- S.9 Graph fractions as decimals on number lines
- S.10 Convert fractions and mixed numbers to decimals - denominators of 10 and 100
- S.12 Convert decimals to fractions and mixed numbers

**4.NF.C.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual model.

- L.3 Compare money amounts
- S.15 Compare decimals using models
- S.16 Compare decimals on number lines
- S.17 Compare decimal numbers
- S.20 Compare decimals and fractions on number lines
- S.21 Compare decimals and fractions

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## 4.MD Measurement and Data

**4.MD.A** Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

**4.MD.A.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

- M.4** Which customary unit is appropriate?
- M.9** Conversion tables - customary units
- M.13** Which metric unit is appropriate?
- M.18** Conversion tables - metric units
- M.19** Convert metric mixed units
- N.1** Convert time units

**4.MD.A.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

- M.2** Measurement word problems
- M.3** Measurement word problems with fractions
- M.10** Compare customary units by multiplying
- N.7** Elapsed time: word problems
- N.8** Find start and end times: multi-step word problems

**4.MD.A.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

- AA.1** Find the perimeter of rectangles using formulas
- AA.4** Perimeter: word problems
- AA.7** Find the area or missing side length of a rectangle
- AA.8** Area: word problems
- AA.10** Area between two rectangles
- AA.12** Relationship between area and perimeter
- AA.13** Area and perimeter: word problems

**AA.16** Use area and perimeter to determine cost

#### 4.MD.B Represent and interpret data.

**4.MD.B.4** Make a line plot to display a data set of measurements in fractions of a unit ( $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

**I.8** Create and interpret line plots with fractions

#### 4.MD.C Geometric measurement: understand concepts of angle and measure angles.

**4.MD.C.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

**4.MD.C.5a** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through  $\frac{1}{360}$  of a circle is called a "one-degree angle," and can be used to measure angles.

**Y.2** Angles as fractions of a circle

**Y.3** Use fractions to find the measure of an angle

**Y.4** Angles of 90, 180, 270, and 360 degrees

**Y.5** Measure angles on a circle

**4.MD.C.5b** An angle that turns through  $n$  one-degree angles is said to have an angle measure of  $n$  degrees.

**Y.3** Use fractions to find the measure of an angle

**Y.4** Angles of 90, 180, 270, and 360 degrees

**Y.8** Estimate angle measurements

**4.MD.C.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

**Y.6** Measure angles with a protractor

**Y.7** Draw angles with a protractor

**4.MD.C.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

- Y.9** Adjacent angles
- Y.10** Angle measures: word problems

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## 4.G Geometry

**4.G.A** Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

**4.G.A.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

- V.3** Points, lines, line segments, rays, and angles
- V.4** Parallel, perpendicular, and intersecting lines
- V.5** Identify parallel, perpendicular, and intersecting lines
- W.4** Parallel sides in quadrilaterals
- Y.1** Acute, right, obtuse, and straight angles

**4.G.A.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

- W.1** Acute, obtuse, and right triangles
- W.** Sides and angles of quadrilaterals
- W.5** Identify parallelograms
- W.6** Identify trapezoids
- W.7** Identify rectangles
- W.8** Identify rhombuses
- W.9** Classify quadrilaterals
- W.10** Pick all the names for a quadrilateral

**4.G.A.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

- X.1** Identify lines of symmetry
- X.2** Draw lines of symmetry
- X.3** Count lines of symmetry

### MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

- E.14 Divide 2-digit numbers by 1-digit numbers: interpret remainders
- E.20 Divide larger numbers by 1-digit numbers: interpret remainders
- F.4 Addition, subtraction, multiplication, and division word problems
- F.6 Word problems with extra or missing information
- F.11 Multi-step word problems
- F.13 Multi-step word problems: identify reasonable answers
- F.14 Choose numbers with a particular sum, difference, product, or quotient
- I.8 Create and interpret line plots with fractions
- N.8 Find start and end times: multi-step word problems
- P.17 Add and subtract fractions with like denominators in recipes
- Y.9 Adjacent angles
- AA.10 Area between two rectangles
- AA.12 Relationship between area and perimeter
- AA.13 Area and perimeter: word problems

## MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

- A.21 Rounding: up to millions place
- D.10 Compare numbers using multiplication
- E.14 Divide 2-digit numbers by 1-digit numbers: interpret remainders
- E.20 Divide larger numbers by 1-digit numbers: interpret remainders
- F.4 Addition, subtraction, multiplication, and division word problems
- M.4 Which customary unit is appropriate?
- M.10 Compare customary units by multiplying
- M.13 Which metric unit is appropriate?
- O.6 Graph equivalent fractions on number lines
- O.19 Compare fractions
- R.6 Multiply unit fractions by whole numbers: word problems
- R.13 Multiply fractions by whole numbers: word problems
- S.17 Compare decimal numbers
- S.21 Compare decimals and fractions

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## MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- L.3** Compare money amounts
- O.5** Find equivalent fractions using area models
- S.16** Compare decimals on number lines
- S.20** Compare decimals and fractions on number lines
- W.5** Identify parallelograms
- W.6** Identify trapezoids
- W.7** Identify rectangles
- W.8** Identify rhombuses

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## MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- B.2** Add two numbers up to five digits: word problems
- B.11** Estimate sums: word problems

- C.2 Subtract numbers up to five digits: word problems
- C.9 Estimate differences: word problems
- D.39 Multiply a 2-digit number by a 2-digit number: word problems
- E.12 Divide 2-digit numbers by 1-digit numbers: word problems
- E.18 Divide larger numbers by 1-digit numbers: word problems
- F.4 Addition, subtraction, multiplication, and division word problems
- F.6 Word problems with extra or missing information
- F.11 Multi-step word problems
- N.7 Elapsed time: word problems
- P.17 Add and subtract fractions with like denominators in recipes
- R.6 Multiply unit fractions by whole numbers: word problems
- R.13 Multiply fractions by whole numbers: word problems
- R.14 Multiply fractions and mixed numbers by whole numbers in recipes
- AA.13 Area and perimeter: word problems

## MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

- B.10 Estimate sums
- C.8 Estimate differences
- D.28 Estimate products: multiply by 1-digit numbers
- D.32 Multiply 2-digit numbers by 2-digit numbers using area models I

- D.33** Multiply 2-digit numbers by 2-digit numbers using area models II
- D.35** Box multiplication
- D.36** Lattice multiplication
- D.38** Multiply a 2-digit number by a 2-digit number
- O.5** Find equivalent fractions using area models
- O.15** Compare fractions using models
- O.19** Compare fractions
- P.7** Add fractions with like denominators using number lines
- P.8** Add fractions with like denominators
- P.11** Subtract fractions with like denominators using number lines
- P.12** Subtract fractions with like denominators
- P.13** Add and subtract fractions with like denominators using number lines
- R.1** Multiply unit fractions by whole numbers using number lines
- R.2** Multiply unit fractions by whole numbers using models
- R.7** Multiply fractions by whole numbers using number lines
- S.16** Compare decimals on number lines
- Y.6** Measure angles with a protractor

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## MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

- A.19** Prime and composite: up to 100
- I.8** Create and interpret line plots with fractions
- M.4** Which customary unit is appropriate?
- M.13** Which metric unit is appropriate?

- W.5 Identify parallelograms
- W.6 Identify trapezoids
- W.7 Identify rectangles
- W.8 Identify rhombuses
- W.9 Classify quadrilaterals
- X.3 Count lines of symmetry
- Y.6 Measure angles with a protractor

## MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

- A.2 Convert between standard and expanded form
- B.6 Properties of addition
- D.25 Distributive property: find the missing factor
- D.35 Box multiplication
- D.36 Lattice multiplication
- K.6 What is true about the given pattern?
- K.7 What is true about the pattern made by the rule?
- P.2 Decompose fractions into unit fractions
- P.4 Decompose fractions multiple ways
- P.8 Add fractions with like denominators
- P.12 Subtract fractions with like denominators

- Q.2 Identify fraction expressions with a particular sum: denominators of 10 and 100
  - Y.9 Adjacent angles
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## MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

- B.5 Addition: fill in the missing digits
- C.5 Subtraction: fill in the missing digits
- D.23 Multiplication patterns over increasing place values
- D.37 Multiply a 2-digit number by a 2-digit number: complete the missing steps
- D.48 Multiplication input/output tables
- E.22 Division patterns over increasing place values
- K.3 Make a repeating pattern
- K.5 Use a rule to complete a number pattern
- S.10 Convert fractions and mixed numbers to decimals - denominators of 10 and 100
- S.12 Convert decimals to fractions and mixed numbers