

5.OA Operations and Algebraic Thinking

5.OA.A Write and interpret numerical expressions.

5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

- O.5** Evaluate numerical expressions
- O.6** Evaluate numerical expressions with parentheses
- O.8** Identify mistakes involving the order of operations

5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

- O.3** Write numerical expressions: one operation
- O.4** Write numerical expressions: two operations

5.OA.B Analyze patterns and relationships.

5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

- S.2** Compare patterns
- U.8** Complete a table for a two-variable relationship
- U.9** Complete a table from a graph
- U.11** Graph a two-variable relationship

5.NBT Number and Operations in Base Ten

5.NBT.A Understand the place value system.

5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.

- A.1** Convert between standard and expanded form
- A.2** Place value
- G.4** Place values in decimal numbers

5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

- E.1** Understanding powers of ten
- E.2** Evaluate powers of ten
- E.3** Write powers of ten with exponents
- E.4** Multiply a whole number by a power of ten: with exponents
- E.5** Multiply a decimal by a power of ten: with exponents
- E.6** Divide by a power of ten: with exponents
- E.7** Multiply and divide by a power of ten: with exponents
- I.3** Multiply a decimal by a power of ten
- I.** Multiply by 0.1 or 0.01
- I.4** Multiply by a power of ten with decimals: find the missing number
- J.1** Divide by powers of ten
- J.2** Decimal division patterns over increasing place values
- J.3** Divide by a power of ten with decimals: find the missing number
- J.** Divide by 0.1 or 0.01

5.NBT.A.3 Read, write, and compare decimals to thousandths.

5.NBT.A.3a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

- G.1** What decimal number is illustrated?

- G.3** Understanding decimals expressed in words
- G.6** Convert decimals between standard and expanded form
- G.19** Convert decimals between standard and expanded form using fractions

5.NBT.A.3b Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

- G.10** Compare decimals using grids
- G.11** Compare decimals on number lines
- G.12** Compare decimal numbers

5.NBT.A.4 Use place value understanding to round decimals to any place.

- G.8** Round decimals
- H.8** Estimate sums and differences of decimals using rounding

5.NBT.B Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

- C.13** Multiply by 2-digit numbers: complete the missing steps
- C.14** Multiply 2-digit numbers by 2-digit numbers
- C.15** Multiply 2-digit numbers by 3-digit numbers
- C.16** Multiply 2-digit numbers by larger numbers
- C.17** Multiply by 2-digit numbers: word problems
- C.18** Multiply three or more numbers up to 2 digits each
- C.19** Multiply by 3-digit numbers
- C.20** Multiply three numbers up to 3 digits each
- C.21** Multiply three or more numbers: word problems

5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- D.7 Divide numbers ending in zeroes
- D.8 Divide numbers ending in zeroes: word problems
- D.12 Divide by 2-digit numbers using models
- D.13 Divide by 2-digit numbers using partial quotients
- D.14 Divide 2-digit and 3-digit numbers by 2-digit numbers
- D.15 Divide 2-digit and 3-digit numbers by 2-digit numbers: word problems
- D.16 Divide 4-digit numbers by 2-digit numbers
- D.17 Divide 4-digit numbers by 2-digit numbers: word problems
- D.19 Multi-step word problems: multiplicative comparison
- D.20 Relate multiplication and division
- D.22 Choose numbers with a particular quotient

5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

- H. Add decimal numbers using blocks
- H.1 Add decimal numbers
- H.2 Subtract decimal numbers
- H.3 Add and subtract decimal numbers
- H.4 Add and subtract decimals: word problems
- H.5 Choose decimals with a particular sum or difference
- H.6 Complete the decimal addition or subtraction sentence
- I.5 Multiply a decimal by a one-digit whole number using blocks
- I.6 Multiply a decimal by a one-digit whole number using the distributive property
- I.7 Multiply a decimal by a one-digit whole number
- I.8 Multiply a decimal by a two-digit whole number using area models
- I.9 Multiply a decimal by a multi-digit whole number
- I.10 Multiply decimals and whole numbers: word problems
- I.11 Multiply three or more numbers, one of which is a decimal
- I.12 Complete the decimal multiplication sentence using grids

- I.13 Multiply decimals using grids
- I.14 Multiply two decimals: where does the decimal point go?
- I.15 Multiply two decimals: products up to hundredths
- I.16 Multiply two decimals: products up to thousandths
- J.4 Divide decimals using blocks: complete the equation
- J.5 Divide decimals using area models: complete the equation
- J.6 Division with decimal quotients
- J.7 Division with decimal quotients and rounding
- J.8 Division with decimal quotients: word problems
- J.10 Divide by decimals without adding zeroes
- J.11 Divide by decimals
- O.11 Add, subtract, multiply, and divide decimals
- O.12 Add, subtract, multiply, and divide decimals: word problems
- R.1 Add and subtract money amounts
- R.2 Add and subtract money: word problems
- R.5 Multiply money amounts: word problems
- R.8 Divide money amounts: word problems
- R.10 Unit prices
- EE.10 Keeping financial records

5.NF Number and Operations—Fractions

5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

- L.6 Add fractions with unlike denominators using models
- L.7 Add up to 4 fractions with denominators of 10 and 100
- L.8 Add fractions with unlike denominators

- L.9 Subtract fractions with unlike denominators using models
- L.11 Subtract fractions with unlike denominators
- L.13 Add 3 or more fractions with unlike denominators
- L.16 Complete addition and subtraction sentences with fractions
- L.19 Add mixed numbers with unlike denominators
- L.20 Subtract mixed numbers with unlike denominators
- L.23 Complete addition and subtraction sentences with mixed numbers

5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

- L.4 Add and subtract fractions with like denominators: word problems
- L.10 Estimate sums and differences of fractions using benchmarks
- L.12 Add and subtract fractions with unlike denominators: word problems
- L.14 Add 3 or more fractions: word problems
- L.21 Add and subtract mixed numbers: word problems
- L.22 Add and subtract fractions in recipes

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

- K.1 Fractions review
- K.2 Fractions of a whole: word problems
- K.16 Relate division and fractions
- K.17 Understand fractions as division: word problems

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.B.4a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

- M.5** Multiply fractions by whole numbers using arrays
- M.11** Multiply fractions by whole numbers I
- M.12** Multiply fractions by whole numbers II
- M.14** Multiply fractions by whole numbers: input/output tables
- M.15** Fractions of a number I
- M.16** Fractions of a number: word problems
- M.** Fractions of a number II
- M.17** Multiply two unit fractions using models
- M.19** Multiply two fractions using models
- M.20** Multiply two fractions
- M.35** Multiply a mixed number by a whole number
- M.36** Multiply a mixed number by a fraction
- M.37** Multiply two mixed numbers

5.NF.B.4b Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

- M.17** Multiply two unit fractions using models
- M.18** Multiply two fractions using models: fill in the missing factor
- M.19** Multiply two fractions using models
- M.29** Understand fraction multiplication and area
- M.30** Multiply fractions to find area
- DD.6** Area of squares and rectangles with fractions
- DD.12** Area and perimeter: word problems

5.NF.B.5 Interpret multiplication as scaling (resizing), by:

5.NF.B.5a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

- M.23** Scaling whole numbers by fractions
- M.24** Scaling fractions by fractions
- M.25** Scaling mixed numbers by fractions

5.NF.B.5b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

- M.19** Multiply two fractions using models
- M.22** Scaling whole numbers by fractions: justify your answer

5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

- M.13** Multiply fractions by whole numbers: word problems
- M.21** Multiply two fractions: word problems
- M.39** Multiplication with mixed numbers: word problems
- M.40** Multiply fractions and mixed numbers in recipes

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.B.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

- N.2** Divide unit fractions by whole numbers
- N.5** Divide unit fractions and whole numbers using area models

5.NF.B.7b Interpret division of a whole number by a unit fraction, and compute such quotients.

- N.3** Divide whole numbers by unit fractions using models
- N.4** Divide whole numbers by unit fractions
- N.5** Divide unit fractions and whole numbers using area models

5.NF.B.7c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

N.7 Divide unit fractions and whole numbers: word problems

5.MD Measurement and Data

5.MD.A Convert like measurement units within a given measurement system.

5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

- Y.2** Compare and convert customary units of length
- Y.3** Compare and convert customary units of weight
- Y.4** Compare and convert customary units of volume
- Y.5** Compare and convert customary units
- Y.6** Conversion tables - customary units
- Y.7** Compare customary units by multiplying
- Y.8** Convert customary units involving fractions
- Y.9** Convert mixed customary units
- Y.10** Add and subtract mixed customary units
- Y.11** Multi-step problems with customary unit conversions
- Y.13** Compare and convert metric units of length
- Y.14** Compare and convert metric units of weight
- Y.15** Compare and convert metric units of volume
- Y.16** Compare and convert metric units
- Y.17** Convert metric units involving decimals
- Y.18** Conversion tables - metric units
- Y.19** Convert metric mixed units
- Y.20** Add and subtract metric mixed units
- Y.21** Multi-step problems with metric unit conversions

Y.22 Multi-step problems with customary or metric unit conversions

5.MD.B Represent and interpret data.

5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots.

V.11 Create and interpret line plots with fractions

5.MD.C Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.C.3a A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

DD.13 Volume of rectangular prisms made of unit cubes

5.MD.C.3b A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

DD.13 Volume of rectangular prisms made of unit cubes

5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

DD.13 Volume of rectangular prisms made of unit cubes

5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

5.MD.C.5a Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

DD.13 Volume of rectangular prisms made of unit cubes

5.MD.C.5b Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

- DD.15** Volume of cubes and rectangular prisms
- DD.17** Volume of cubes and rectangular prisms: word problems
- DD.18** Compare volumes and dimensions of rectangular prisms: word problems

5.MD.C.5c Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

- DD.** Volume of compound figures

5.G Geometry

5.G.A Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

- T.1** Describe the coordinate plane
- T.2** Objects on a coordinate plane

5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

- T.3** Graph points on a coordinate plane
- T.4** Graph points from a table
- T.7** Coordinate planes as maps
- T.8** Follow directions on a coordinate plane

5.G.B Classify two-dimensional figures into categories based on their properties.

5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

- AA.4** Parallel sides in quadrilaterals
- AA.5** Identify parallelograms
- AA.7** Identify rectangles
- AA.8** Identify rhombuses
- AA.9** Classify quadrilaterals
- AA.11** Identify the relationships between quadrilaterals
- AA.12** Describe relationships among quadrilaterals

5.G.B.4 Classify two-dimensional figures in a hierarchy based on properties.

- Z.1** Is it a polygon?
- Z.3** Regular and irregular polygons
- Z.9** Sort polygons into Venn diagrams
- AA.1** Acute, obtuse, and right triangles
- AA.2** Scalene, isosceles, and equilateral triangles
- AA.3** Classify triangles
- AA.5** Identify parallelograms
- AA.7** Identify rectangles
- AA.8** Identify rhombuses
- AA.9** Classify quadrilaterals

MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

- C.21 Multiply three or more numbers: word problems
- D.22 Choose numbers with a particular quotient
- H.4 Add and subtract decimals: word problems
- L.4 Add and subtract fractions with like denominators: word problems
- L.12 Add and subtract fractions with unlike denominators: word problems
- L.14 Add 3 or more fractions: word problems
- L.21 Add and subtract mixed numbers: word problems
- R.2 Add and subtract money: word problems
- R.10 Unit prices
- V.11 Create and interpret line plots with fractions
- Y.22 Multi-step problems with customary or metric unit conversions
- DD.12 Area and perimeter: word problems

MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

- G.1** What decimal number is illustrated?
- G.8** Round decimals
- G.11** Compare decimals on number lines
- H.4** Add and subtract decimals: word problems
- L.4** Add and subtract fractions with like denominators: word problems
- L.12** Add and subtract fractions with unlike denominators: word problems
- M.18** Multiply two fractions using models: fill in the missing factor
- M.21** Multiply two fractions: word problems
- M.23** Scaling whole numbers by fractions
- M.24** Scaling fractions by fractions
- M.25** Scaling mixed numbers by fractions
- N.7** Divide unit fractions and whole numbers: word problems
- O.4** Write numerical expressions: two operations
- U.8** Complete a table for a two-variable relationship
- Y.5** Compare and convert customary units
- Y.16** Compare and convert metric units

MP3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- G.11** Compare decimals on number lines
- M.22** Scaling whole numbers by fractions: justify your answer
- M.29** Understand fraction multiplication and area
- S.2** Compare patterns
- Y.7** Compare customary units by multiplying
- AA.5** Identify parallelograms
- AA.7** Identify rectangles
- AA.8** Identify rhombuses

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

- C.17** Multiply by 2-digit numbers: word problems
- D.15** Divide 2-digit and 3-digit numbers by 2-digit numbers: word problems

- H.4 Add and subtract decimals: word problems
- I.10 Multiply decimals and whole numbers: word problems
- J.8 Division with decimal quotients: word problems
- L.14 Add 3 or more fractions: word problems
- L.21 Add and subtract mixed numbers: word problems
- M.13 Multiply fractions by whole numbers: word problems
- M.21 Multiply two fractions: word problems
- M.39 Multiplication with mixed numbers: word problems
- N.7 Divide unit fractions and whole numbers: word problems
- R.5 Multiply money amounts: word problems
- R.8 Divide money amounts: word problems
- T.2 Objects on a coordinate plane
- U.8 Complete a table for a two-variable relationship
- DD.12 Area and perimeter: word problems

MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

- G.11 Compare decimals on number lines
- H.8 Estimate sums and differences of decimals using rounding
- I.12 Complete the decimal multiplication sentence using grids
- I.15 Multiply two decimals: products up to hundredths

- L.6 Add fractions with unlike denominators using models
- L.8 Add fractions with unlike denominators
- L.9 Subtract fractions with unlike denominators using models
- L.11 Subtract fractions with unlike denominators
- L.19 Add mixed numbers with unlike denominators
- L.20 Subtract mixed numbers with unlike denominators
- M.17 Multiply two unit fractions using models
- M.19 Multiply two fractions using models
- M.20 Multiply two fractions
- N.3 Divide whole numbers by unit fractions using models
- N.6 Divide whole numbers and unit fractions
- T.8 Follow directions on a coordinate plane
- U.9 Complete a table from a graph
- U.11 Graph a two-variable relationship

MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

- I.3 Multiply a decimal by a power of ten
- J.6 Division with decimal quotients
- J.7 Division with decimal quotients and rounding
- M.23 Scaling whole numbers by fractions
- M.24 Scaling fractions by fractions
- M.25 Scaling mixed numbers by fractions
- T.2 Objects on a coordinate plane

- AA.3** Classify triangles
 - AA.9** Classify quadrilaterals
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MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

- G.6** Convert decimals between standard and expanded form
 - G.19** Convert decimals between standard and expanded form using fractions
 - L.7** Add up to 4 fractions with denominators of 10 and 100
 - L.16** Complete addition and subtraction sentences with fractions
 - L.23** Complete addition and subtraction sentences with mixed numbers
 - S.2** Compare patterns
 - U.8** Complete a table for a two-variable relationship
 - DD.13** Volume of rectangular prisms made of unit cubes
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MP8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

- C.13** Multiply by 2-digit numbers: complete the missing steps
- I.3** Multiply a decimal by a power of ten
- J.1** Divide by powers of ten
- J.2** Decimal division patterns over increasing place values
- M.14** Multiply fractions by whole numbers: input/output tables
- M.23** Scaling whole numbers by fractions
- M.24** Scaling fractions by fractions
- M.25** Scaling mixed numbers by fractions