7.RP Ratios and Proportional Relationships

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

- J.6 Calculate unit rates with fractions
- M.3 Unit prices

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

- J.2 Identify equivalent ratios
- J.4 Equivalent ratios: word problems
- J.9 Do the ratios form a proportion?
- J.10 Do the ratios form a proportion: word problems
- K.3 Identify proportional relationships by graphing
- K.6 Identify proportional relationships from graphs and equations
- K.7 Identify proportional relationships from tables

7.RP.A.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

- K.1 Find the constant of proportionality from a table
- K.4 Find the constant of proportionality from a graph

7.RP.A.2c Represent proportional relationships by equations.
7.RP.A.2d Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where $r$ is the unit rate.

K.10 Interpret graphs of proportional relationships

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems.

7.NS The Number System
7.NS.A Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.1a Describe situations in which opposite quantities combine to make 0.

B.4 Absolute value and opposite integers
B.5 Quantities that combine to zero: word problems

7.NS.A.1b Understand \( p + q \) as the number located a distance \(|q|\) from \( p \), in the positive or negative direction depending on whether \( q \) is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

C.2 Add integers using number lines
C.3 Integer addition rules
C.10 Integer addition and subtraction rules
H.13 Apply addition and subtraction rules

7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, \( p - q = p + (-q) \). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

C.7 Subtract integers using number lines
C.8 Integer subtraction rules
C.10 Integer addition and subtraction rules
H.13 Apply addition and subtraction rules

7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.

C.1 Add integers using counters
C.4 Add integers
C.6 Subtract integers using counters
C.9 Subtract integers
7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

7.NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as \((-1)(-1) = 1\) and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

7.NS.A.2b Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If \(p\) and \(q\) are integers, then \(-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}\). Interpret quotients of rational numbers by describing real-world contexts.

7.NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.
7.NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

H.1 Convert fractions or mixed numbers to decimals

H. Classify rational numbers

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.
7.EE Expressions and Equations

7.EE.A Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

- R. Simplify expressions by combining like terms
- R.11 Multiply using the distributive property: area models
- R.12 Multiply using the distributive property
- R.13 Write equivalent expressions using properties
- R.14 Add and subtract linear expressions
- R. Factor linear expressions: area models
- R.16 Factors of linear expressions
- R.17 Identify equivalent linear expressions using algebra tiles
- R.18 Identify equivalent linear expressions I
- R.19 Identify equivalent linear expressions II

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

- R.20 Identify equivalent linear expressions: word problems
7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

- C.25 Evaluate numerical expressions involving integers
- E.10 Maps with decimal distances
- E.11 Evaluate numerical expressions involving decimals
- G.17 Maps with fractional distances
- G.18 Evaluate numerical expressions involving fractions
- I.7 Evaluate numerical expressions involving exponents
- N.2 Multi-step word problems
- R.4 Evaluate linear expressions
- R.5 Evaluate multi-variable expressions
- R.7 Evaluate nonlinear expressions

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

7.EE.B.4a Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

- S.4 Model and solve equations using algebra tiles
- S. Solve two-step equations without parentheses
- S. Solve two-step equations with parentheses
- S.7 Solve two-step equations
- S. Choose two-step equations: word problems
- S.8 Solve two-step equations: word problems
- S.9 Solve equations involving like terms
- S.10 Solve equations: complete the solution
- U.4 Evaluate two-variable equations: word problems
7.EE.B.4b Solve word problems leading to inequalities of the form \( px + q > r \) or \( px + q < r \), where \( p, q, \) and \( r \) are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

- **T.4** Solve one-step inequalities
- **T.5** Graph solutions to one-step inequalities
- **T.6** One-step inequalities: word problems
- **T.7** Solve two-step inequalities
- **T.8** Graph solutions to two-step inequalities

**7.G Geometry**

**7.G.A** Draw, construct, and describe geometrical figures and describe the relationships between them.

- **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
  - **Z.1** Scale drawings: word problems
  - **Z.2** Scale drawings: scale factor word problems
  - **Z.3** Perimeter and area: changes in scale

- **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
  - **W.4** Triangle inequality
  - **W.8** Graph triangles and quadrilaterals

- **7.G.A.3** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
  - **X.4** Cross sections of three-dimensional figures
7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

- W.22 Parts of a circle
- Y.5 Area of circles
- Y.6 Circumference of circles
- Y.7 Circles: word problems

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

- W.16 Identify complementary, supplementary, vertical, and adjacent angles
- W.17 Find measures of complementary, supplementary, vertical, and adjacent angles

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- Y.2 Area of rectangles and parallelograms
- Y.3 Area of triangles and trapezoids
- Y.8 Volume of cubes and prisms
- Y.9 Volume of cubes and rectangular prisms: word problems
- Y.12 Surface area of cubes and prisms
- Y.17 Area of compound figures with triangles
- Y.19 Area between two shapes

7.SP Statistics and Probability

7.SP.A Use random sampling to draw inferences about a population.
7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

DD.8 Identify representative, random, and biased samples

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

J.13 Estimate population size using proportions

7.SP.B Draw informal comparative inferences about two populations.

7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

DD.1 Calculate mean, median, mode, and range
DD.2 Interpret charts and graphs to find mean, median, mode, and range
DD.3 Mean, median, mode, and range: find the missing number
DD.4 Changes in mean, median, mode, and range

7.SP.C Investigate chance processes and develop, use, and evaluate probability models.

7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

EE.1 Probability of simple events
7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

EE.6 Make predictions using theoretical probability

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.C.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

EE.1 Probability of simple events
EE.2 Probability of simple events and opposite events

7.SP.C.7b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

EE.4 Experimental probability

7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.C.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

EE.9 Probability of compound events
EE.11 Probability of independent and dependent events

7.SP.C.8b Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

EE.7 Compound events: find the number of outcomes
EE.8 Compound events: find the number of sums
EE.12 Find the number of outcomes: word problems

7.SP.C.8c Design and use a simulation to generate frequencies for compound events.
EE. Which simulation represents the situation?
MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

- C.14 Add and subtract integers: word problems
- E.8 Add, subtract, multiply, and divide decimals: word problems
- G.16 Add, subtract, multiply, and divide fractions and mixed numbers: word problems
- J.12 Solve proportions: word problems
- J.13 Estimate population size using proportions
- L.9 Solve percent equations: word problems
- L.11 Percent of change: word problems
- M.1 Add, subtract, multiply, and divide money amounts: word problems
- M.5 Unit prices: find the total price
- M.8 Find the percent: tax, discount, and more
- M.10 Multi-step problems with percents
- M.12 Simple interest
- Y.7 Circles: word problems
- Z.1 Scale drawings: word problems
- DD.2 Interpret charts and graphs to find mean, median, mode, and range
MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

- H.18 Apply multiplication and division rules
- K.1 Find the constant of proportionality from a table
- K.2 Write equations for proportional relationships from tables
- K.4 Find the constant of proportionality from a graph
- K.5 Write equations for proportional relationships from graphs
- K.10 Interpret graphs of proportional relationships
- K.11 Write and solve equations for proportional relationships
- M.4 Unit prices with unit conversions
- R.4 Evaluate linear expressions
- R.5 Evaluate multi-variable expressions
- R.7 Evaluate nonlinear expressions
- S.8 Solve two-step equations: word problems
- U.4 Evaluate two-variable equations: word problems

MP3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

J.2 Identify equivalent ratios
J.4 Equivalent ratios: word problems
J.9 Do the ratios form a proportion?
J.10 Do the ratios form a proportion: word problems
K.3 Identify proportional relationships by graphing
R.13 Write equivalent expressions using properties
S.10 Solve equations: complete the solution
U.2 Identify independent and dependent variables
DD.8 Identify representative, random, and biased samples
EE.6 Make predictions using theoretical probability

MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
## MP5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MP6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

C.13 Complete addition and subtraction equations with integers
C.23 Complete multiplication and division equations with integers
H.3 Convert between decimals and fractions or mixed numbers
H.11 Add and subtract rational numbers
H.17 Multiply and divide rational numbers
K.6 Identify proportional relationships from graphs and equations
R.12 Multiply using the distributive property
R.14 Add and subtract linear expressions
R.16 Factors of linear expressions
R.19 Identify equivalent linear expressions II
S.9 Solve equations involving like terms
W.8 Graph triangles and quadrilaterals
Y.18 Area of compound figures with triangles, semicircles, and quarter circles
Y.19 Area between two shapes
EE.7 Compound events: find the number of outcomes

MP8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation \((\frac{y}{x} - 2)/(\frac{y}{x} - 1) = 3\). Noticing the regularity in the way terms cancel when expanding \((\frac{x}{x} - 1)(\frac{x}{x} + 1), (\frac{x}{x} - 1)(\frac{x}{x}^{2} + \frac{x}{x} + 1), \) and \((\frac{x}{x} - 1)(\frac{x}{x}^{3} + \frac{x}{x}^{2} + \frac{x}{x} + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

- **C.10** Integer addition and subtraction rules
- **C.21** Integer multiplication and division rules
- **I.7** Evaluate numerical expressions involving exponents
- **K.1** Find the constant of proportionality from a table
- **K.4** Find the constant of proportionality from a graph