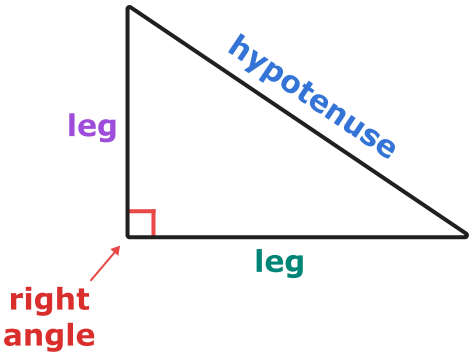
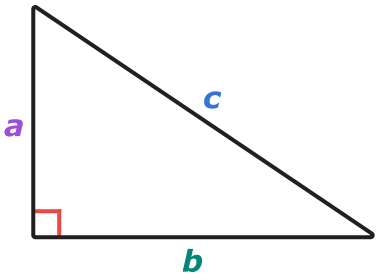
The Pythagorean theorem

The **Pythagorean theorem** describes a special relationship between the sides of a right triangle.

A right triangle is made up of two **legs** and a **hypotenuse**. The legs meet at a 90° angle. The hypotenuse is the side opposite the right angle. The hypotenuse is always the longest side. We use ***a*** and ***b*** to represent the lengths of the legs, and ***c*** to represent the length of the hypotenuse.

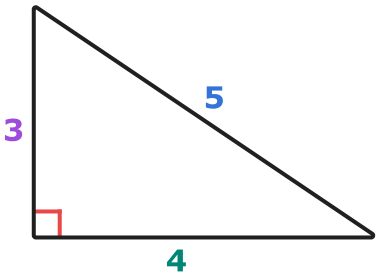




According to the Pythagorean theorem, the square of the hypotenuse is equal to the sum of the squares of the legs. You can show that relationship with the following equation.

$$a^2 + b^2 = c^2$$

Let's look at an example. The right triangle below has a leg with a length of **3**, a leg with a length of **4**, and a hypotenuse with a length of **5**.

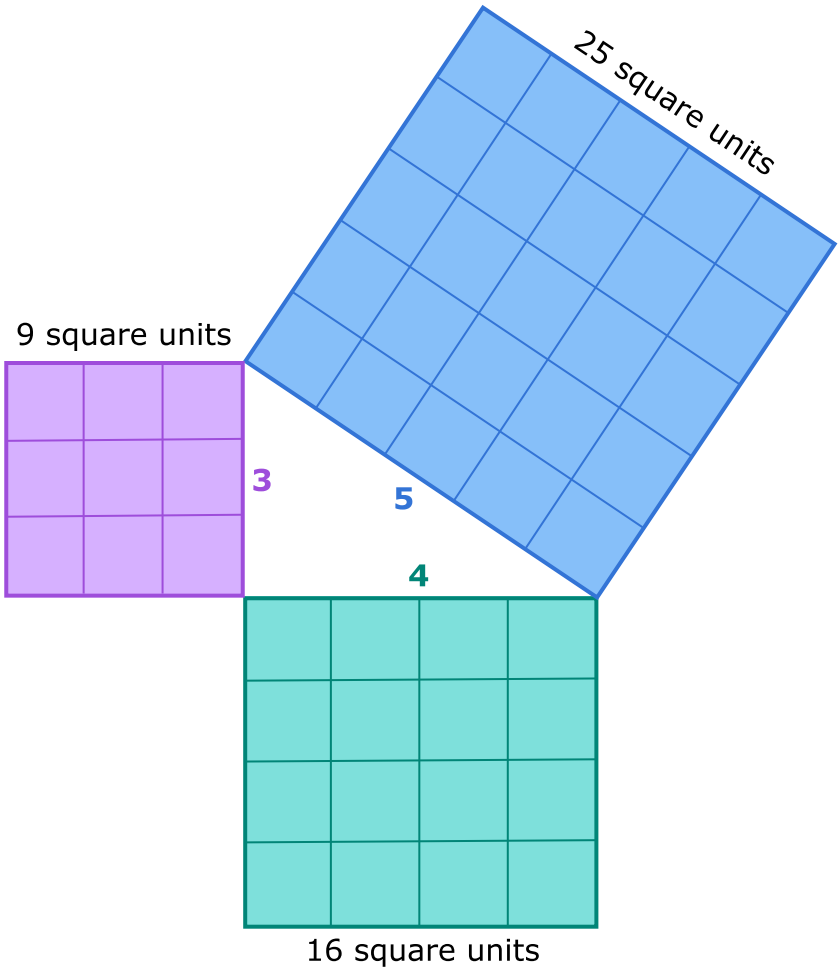


Plug the numbers into the equation.

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

The sum of the squares of the sides equals the sum of the square of the hypotenuse!

You can show this relationship visually. If you make a square on each side of the triangle, the areas of the smaller squares add up to the area of the largest square.



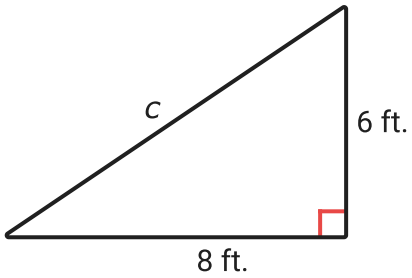
Find missing side lengths

If you know the lengths of any two sides of a right triangle, you can use the Pythagorean theorem to find the length of the third side.

Finding a missing hypotenuse length

You can use the Pythagorean theorem to find the length of the hypotenuse in a right triangle if you know the lengths of the two legs.

Let's try it! Find *c*.



The lengths of the legs are 6 feet and 8 feet. So, you can let $a = 6$ and $b = 8$. Plug in those values for a and b, and simplify.

$$a^2 + b^2 = c^2$$

$$6^2 + 8^2 = c^2$$

$$36 + 64 = c^2$$

$$100 = c^2$$

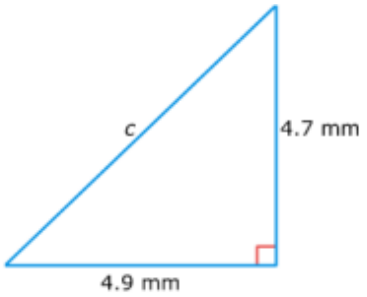
To solve for c, you'll need to find the square root of 100.

$$\sqrt{100} = \sqrt{c^2}$$

$$10 = c$$

So, the hypotenuse is 10 feet long!

Go to IXL to try some practice problems!



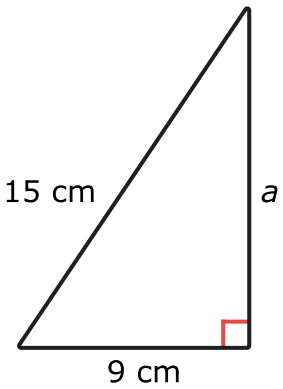
What is the length of the hypotenuse? If necessary, round to the nearest tenth.

c = ____ millimeters

Pythagorean theorem: find the length of the hypotenuse 7ZL

Finding a missing leg length

You can use the Pythagorean theorem to find the length of a leg in a right triangle if you know the lengths of the hypotenuse and other leg.



The length of one of the legs is 9 centimeters, and the length of the hypotenuse is 15 centimeters. So, you can let $b = 9$ and $c = 15$. Plug in these values for b and c, and simplify.

$$a^2 + b^2 = c^2$$

$$a^2 + 9^2 = 15^2$$

$$a^2 + 81 = 225$$

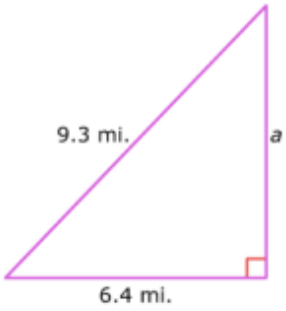
$$a^2 = 144$$

$$\sqrt{a^2} = \sqrt{144}$$

$$a = 12$$

So, the missing leg is 12 centimeters long!

Go to IXL to try some practice problems!



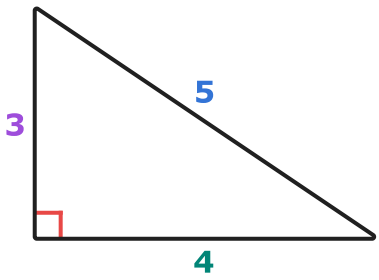
What is the length of the missing leg? If necessary, round to the nearest tenth.

$a =$ ____ miles

Pythagorean theorem: find the missing leg length Y9C

Pythagorean triples

A Pythagorean triple is a set of three positive integers ***a***, ***b***, and ***c*** that make the equation $a^2 + b^2 = c^2$ true. You will often see Pythagorean triples written as (***a***, ***b***, ***c***). When a triangle's sides are a Pythagorean triple, it is a right triangle. For example, (**3**, **4**, **5**) is a Pythagorean triple.



This list shows some common Pythagorean triples.

(**3**, **4**, **5**) (**5**, **12**, **13**) (**7**, **24**, **25**)
(**8**, **15**, **17**) (**9**, **40**, **41**) (**11**, **60**, **61**)

Fun Fact

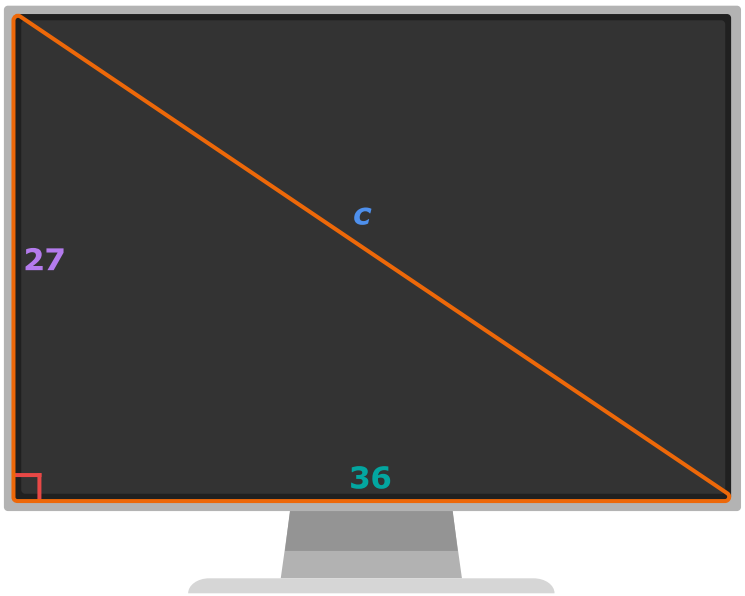
If you multiply all three numbers in a Pythagorean triple by the same integer, you get another Pythagorean triple! For example, if you multiply each number in the Pythagorean triple (**3**, **4**, **5**) by 3, you get the triple (**9**, **12**, **15**).

Real-world applications

Although the Pythagorean theorem has been around for thousands of years, it is still useful for solving everyday problems!

> *Imagine that your friend is selling his old TV. He told you that the TV is 36 inches by 27 inches. You want to compare your friend's TV to new TVs of the same size, but the TVs you find give screen size by diagonal length. How can you use the Pythagorean theorem to calculate the diagonal length of the TV?*

We have the base and height of the TV, and we are looking for the diagonal. We can think of the base, height, and diagonal of the TV as a right triangle and then use the Pythagorean theorem.



So, $a = 27$ and $b = 36$, and c is the length of the diagonal.

$$a^2 + b^2 = c^2$$

$$27^2 + 36^2 = c^2$$

$$729 + 1{,}296 = c^2$$

$$2{,}025 = c^2$$

$$\sqrt{2{,}025} = \sqrt{c^2}$$

$$45 = c$$

So, the TV has a 45-inch diagonal!

Fun Fact

Oceanographers use the Pythagorean theorem to calculate the speed of sound waves in water. Pilots use it to determine when the plane should start descending towards the airport. Surveyors use it to calculate the steepness of hills and mountains. The Pythagorean theorem has many uses!

Go to IXL to try some practice problems!

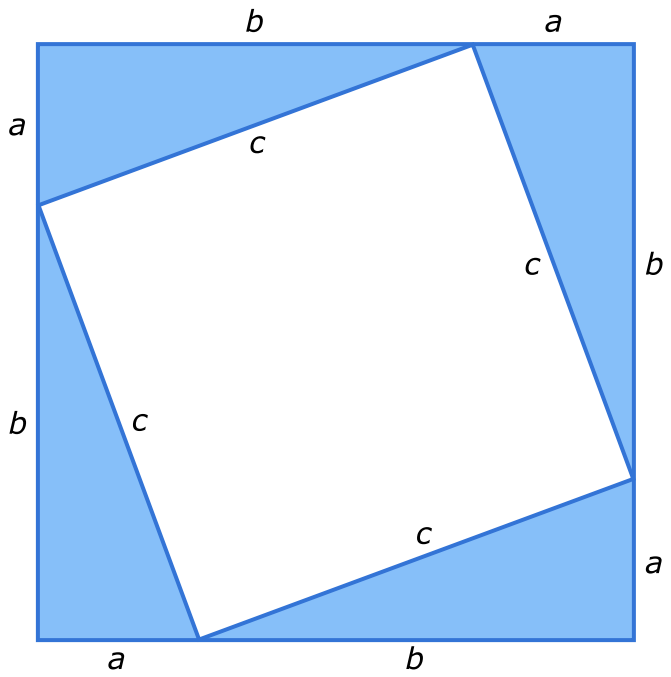
On the school playground, the slide is due west of the tire swing and due south of the monkey bars. If the distance between the slide and the tire swing is 7 feet and the distance between the tire swing and the monkey bars is 10 feet, how far is the slide from the monkey bars? If necessary, round to the nearest tenth.

feet

Pythagorean theorem: word problems 87U

Why does the Pythagorean theorem work?

You can use algebra to prove the Pythagorean theorem.



The diagram shows four right triangles and a small square arranged to form a large square.

You can write the area of the large square in two ways.

First, you can multiply its side lengths. Each side has a length of $a + b$.

$(a + b)(a + b)$

Second, you can add the areas of the shapes within the large square. The area of the small square is c^2. The area of each triangle is $\left(\frac{1}{2}\right)ab$.

$$c^2 + \left(\frac{1}{2}\right)ab + \left(\frac{1}{2}\right)ab + \left(\frac{1}{2}\right)ab + \left(\frac{1}{2}\right)ab$$

$$c^2 + 4 \cdot \left(\frac{1}{2}\right)ab$$

$$c^2 + 2ab$$

You've shown that the area of the large square is $(a + b)(a + b)$ and also $c^2 + 2ab$. Set these areas equal to one another.

$$(a + b)(a + b) = c^2 + 2ab$$

Then, expand the left side of the equation.

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

Now, subtract $2ab$ from both sides.

$$a^2 + b^2 = c^2$$

This proves the Pythagorean theorem!

The converse of the Pythagorean theorem

The Pythagorean theorem also works in reverse. The converse of the Pythagorean theorem says that if the three sides of a triangle satisfy $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Let's try it! Determine if a triangle with side lengths of 6, 10, and 12 is a right triangle.

Plug the side lengths into the equation $a^2 + b^2 = c^2$ and see if the result is true. Use the largest number, 12, as c. It does not matter which length you pick to be a and b.

$$6^2 + 10^2 = 12^2$$

$$36 + 100 = 144$$

$$136 = 144$$

This statement is not true! So, a triangle with sides lengths of 6, 10, and 12 is **not** a right triangle.

Go to IXL to try some practice problems!

A triangle has sides with lengths of 48 inches, 64 inches, and 80 inches. Is it a right triangle?

yes | no

Converse of the Pythagorean theorem: is it a right triangle? JVX

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Skills

Pythagorean theorem: find the length of the hypotenuse 7ZL

Pythagorean theorem: find the missing leg length Y9C

Pythagorean theorem: find the missing leg or hypotenuse length MTM

Pythagorean theorem: find the perimeter VGE

Pythagorean theorem: word problems 87U

Lessons

The converse of the Pythagorean theorem